

Name: _____

Date: _____

SEQUENCES COMMON CORE ALGEBRA II



In Common Core Algebra I, you studied **sequences**, which are ordered lists of numbers. Sequences are extremely important in mathematics, both theoretical and applied. A **sequence** is formally defined as **a function that has as its domain the set the set of positive integers**, i.e. $\{1, 2, 3, \dots, n\}$.

Exercise #1: A sequence is defined by the equation $a(n) = 2n - 1$.

- (a) Find the first three terms of this sequence,
denoted by a_1 , a_2 , and a_3 .
- (b) Which term has a value of 53?

- (c) Explain why there will not be a term that has a value of 70.

Recall that sequences can also be described by using **recursive definitions**. When a sequence is defined recursively, terms are found by operations on previous terms.

Exercise #2: A sequence is defined by the recursive formula: $f(n) = f(n-1) + 5$ with $f(1) = -2$.

- (a) Generate the first five terms of this sequence.
Label each term with proper **function** notation.
- (b) Determine the value of $f(20)$. Hint – think
about how many times you have added 5 to -2 .

Exercise #3: Determine a recursive definition, in terms of $f(n)$, for the sequence shown below. Be sure to include a starting value.

5, 10, 20, 40, 80, 160, ...

Exercise #4: For the recursively defined sequence $t_n = (t_{n-1})^2 + 2$ and $t_1 = 2$, the value of t_4 is

- (1) 18 (3) 456
(2) 38 (4) 1446



Exercise #5: One of the most well-known sequences is the Fibonacci, which is defined recursively using two previous terms. Its definition is given below.

$$f(n) = f(n-1) + f(n-2) \text{ and } f(1) = 1 \text{ and } f(2) = 1$$

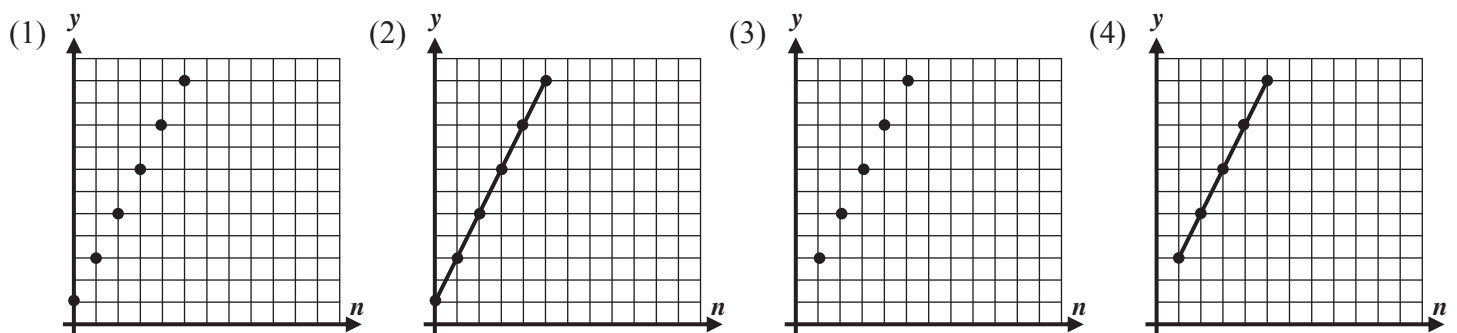
Generate values for $f(3)$, $f(4)$, $f(5)$, and $f(6)$ (in other words, then next four terms of this sequence).

It is often possible to find algebraic formulas for simple sequence, and this skill should be practiced.

Exercise #6: Find an algebraic formula $a(n)$, similar to that in *Exercise #1*, for each of the following sequences. Recall that the domain that you map from will be the set $\{1, 2, 3, \dots, n\}$.

- (a) 4, 5, 6, 7, ... (b) 2, 4, 8, 16, ... (c) $5, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \dots$
- (d) -1, 1, -1, 1, ... (e) 10, 15, 20, 25, ... (f) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \dots$

Exercise #7: Which of the following would represent the graph of the sequence $a_n = 2n + 1$? Explain your choice.



Explanation:



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SEQUENCES

COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Given each of the following sequences defined by formulas, determine and label the first four terms. A variety of different notations is used below for practice purposes.

(a) $f(n) = 7n + 2$ (b) $a_n = n^2 - 5$ (c) $t(n) = \left(\frac{2}{3}\right)^n$ (d) $t_n = \frac{1}{n+1}$

2. Sequences below are defined recursively. Determine and label the **next** three terms of the sequence.

(a) $f(1) = 4$ and $f(n) = f(n-1) + 8$ (b) $a(n) = a(n-1) \cdot \frac{1}{2}$ and $a(1) = 24$

(c) $b_n = b_{n-1} + 2n$ with $b_1 = 5$ (d) $f(n) = 2f(n-1) - n^2$ and $f(1) = 4$

3. Given the sequence 7, 11, 15, 19, ..., which of the following represents a formula that will generate it?

(1) $a(n) = 4n + 7$ (3) $a(n) = 3n + 7$

(2) $a(n) = 3n + 4$ (4) $a(n) = 4n + 3$

4. A recursive sequence is defined by $a_{n+1} = 2a_n - a_{n-1}$ with $a_1 = 0$ and $a_2 = 1$. Which of the following represents the value of a_5 ?

(1) 8 (3) 3

(2) -7 (4) 4

5. Which of the following formulas would represent the sequence 10, 20, 40, 80, 160, ...

(1) $a_n = 10^n$ (3) $a_n = 5(2)^n$

(2) $a_n = 10(2)^n$ (4) $a_n = 2n + 10$



6. For each of the following sequences, determine an algebraic formula, similar to *Exercise #4*, that defines the sequence.

(a) 5, 10, 15, 20, ...

(b) 3, 9, 27, 81, ...

(c) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$

7. For each of the following sequences, state a recursive definition. Be sure to include a starting value or values.

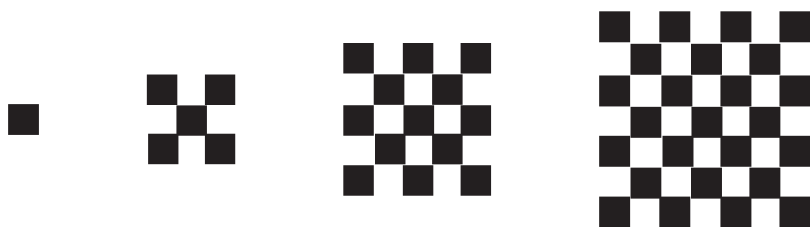
(a) 8, 6, 4, 2, ...

(b) 2, 6, 18, 54, ...

(c) 2, -2, 2, -2, ...

APPLICATIONS

8. A tiling pattern is created from a single square and then expanded as shown. If the number of squares in each pattern defines a sequence, then determine the number of squares in the seventh pattern. Explain how you arrived at your choice. Can you write a recursive definition for the pattern?



REASONING

9. Consider a sequence defined similarly to the Fibonacci, but with a slight twist:

$$f(n) = f(n-1) - f(n-2) \text{ with } f(1) = 2 \text{ and } f(2) = 5$$

Generate terms $f(3)$, $f(4)$, $f(5)$, $f(6)$, $f(7)$, and $f(8)$. Then, determine the value of $f(25)$.

